Experimental validation of a model for the dynamic analysis of gear pumps

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Abstract

This paper mainly concerns the experimental validation of an elastodynamic model of an external gear pump for steering systems. The model takes into account the most important phenomena involved in the operation of this kind of machines. Two main sources of noise and vibration can be considered: pressure and gear meshing. An experimental apparatus has been set up for the measurements of the case accelerations and force components in operation conditions. The model was validated by comparison between simulations and experimental results concerning forces and moments: it deals with the external and inertia components acting on the gears, estimated by the model, and the reactions and inertia components on the pump case and the test plate, obtained by measurements. The validation is carried out comparing the level of the time synchronous average in the time domain and the waterfall maps in the frequency domain, with particular attention to identify system resonances. The validation results are globally satisfactory.

1 Introduction

Nowadays, noise has become one of the most restrictive key feature required for new component designs especially in the automotive industry but also in other fields. Final manufacturers define more restrictive performances that component suppliers should verify. An increment on the resources on design and testing tasks is required from the component suppliers in order to achieve the goals defined by the final customer. This environment increases the interest on modelling the dynamic behaviour of mechanical systems as a way to improve the initial design reducing testing efforts. A good dynamic model could be an useful and powerful tool for the identification of noise and vibration sources and design improvement.

Model development is not an easy work requiring in a first stage a good analysis of the system in order to define the model goals. That means to define the most important phenomena and assumptions that must be included. A second but also important stage in the model development is the formulation or modification of the available theories in order to fulfil the model goals taking into account simultaneously the numerical problems involved. In this way the resulting model should be simple but at the same time provides a prediction of the dynamic behaviour accurate and
reliable enough. Finally the model developed must be verified and validated using experiments. A good model should be a good representation of the real machine and therefore should have a good correspondence with the experimental measurements. During this last task, model parameters should be modified in order to approach real and simulated behaviour, improving the features of the model. This could be also a critical task because good experiments are as rare as good theories and complex systems sometimes introduce additional difficulties associated with the availability of measurement data directly comparable to model output.

This work deals with external gear pumps for automotive applications, which operate at high speed and low pressure. In this situation gear pumps have two main noise sources both of them sharing the same fundamental period: fluid borne noise, as a consequence of the flow from the low to the high pressure chambers, and mechanical noise due to the gear meshing. Due to the complex nature and the combination of both excitation sources, design is a difficult task requiring a great number of experimental tests. Then, a numerical model should be able to predict the effects of design modifications reducing the number of tests required for prototype development and design improvement.

In this paper a non-linear lumped kineto-elastodynamic model for the prediction of the dynamic behaviour of external gear pumps is described. The model initially developed [1] had six degrees of freedom and included the main important phenomena involved in the pump operation as time-varying oil pressure distribution on gears, time-varying meshing stiffness and hydrodynamic journal bearings forces. Subsequently case wear [2], backlash as well as profile errors [3] were added, improving the model features. It is worth noting that all these important effects are included in the same model, in order to take into account their interactions. In this way the resulting model is an useful tool for studying the effects of design parameters concerning both gears (profile shape and errors) and the geometry of other components (input and output chambers, transportation arc, lateral plates grooves, etc). A specific experimental set-up and procedure was developed in order to validate the model [4]. This procedure is based on the comparison between simulations and experimental results concerning forces and moments: it deals with the external and inertia components acting on the gears, estimated by the model, and the reactions and inertia components on the pump case and the test plate, obtained by means of measurements. These are the components that can excite case vibrations and produce noise; consequently, their estimation has a practical interest for noise reduction. In this work the validation is carried out comparing the level of the time synchronous average in the time domain and the waterfall maps in the frequency domain, with particular attention to identify system resonances.

The structure of this paper is the following. In first place the system under study is described highlighting the most important design features that define the dynamic behaviour. Then, the proposed model is shortly described as well as the computational procedure implemented. After the experimental set-up and the test carried out are presented. Following the validation procedure and the results obtained are discussed.

2 External Gear Pump Design and Model

External gear pumps are popular pumping components well suited for handling viscous fluids such as fuel and lubricating oils. They are simple and robust devices that can work at a wide range of pressure and rotational speed providing at the same time a high reliability. Their main applications can be found as lubrication pumps in machine tools, as oil pumps in engines or in fluid power transfer units. Depending on the application several designs are available, nevertheless the most usual configuration uses twin gears assembled in a couple of lateral floating brushes packed inside a case (see Figure 1). This is the type of pump studied in this work. Gear 1 is driven by an electrical
motor through an Oldham coupling allowing small misalignments between their shafts. This coupling is required because the gear shaft centre describes an orbit around a different centroid, depending on the working pressure and rotational speed (in the case under study pressure ranges from 20 to 90 bar and rotational speed from 1000 to 3400 r.p.m.). Lateral floating brushes act as seals for the lateral ends and as supports for gear shafts by means of two hydrodynamic bearings.

During pump operation, at the input port an isolated oil volume is created defined by the space bounded between two subsequent gear teeth, the case and the lateral floating brushes. As the gears turn this isolated volume is carried out toward the output port progressively increasing its pressure. In the gear meshing area, when two tooth pairs become in contact, a trapped volume would arise that would undergo a sudden volume reduction and consequently a violent change in its pressure. To avoid this, relief grooves are present on each floating brush with the aim to connect the trapped volume with the high or the low pressure chambers, avoiding in this way the pressure rise and reducing the noise level.

The main important phenomena involved in the pump dynamic behaviour have been included in the numerical model: pressure distribution variation, meshing forces and hydrodynamic bearing reactions. A scheme of the model is shown in Figure 2. It is a planar model with 6 degrees of freedom \((x_1, y_1, \theta_1, x_2, y_2, \theta_2)\). Two reference frames are used, one for each gear, having their origins coincident with the centres of the gears, the X-axis perpendicular to one of the lines of action and the Y-axis parallel (see Figure 2 (a)). The model input is the coordinate \(\theta_0\), representing the angular displacement of the electric drive, assumed to rotate as constant speed. Coordinate \(\theta_0\) is connected to gear 1 by a torsional spring-damper element being \(K_T\) the torsional shaft stiffness and \(C_T\) the damping. Meshing stiffness and damping, profile errors (obtained from metrological measurements) as well as lubricant squeeze are included in each one of the contact blocks representing the Direct (DLA) and Inverse (ILA) lines of action [3]. The backlash between meshing teeth is also included, as well as the possibility of tooth contact on both contact lines. Meshing stiffness is obtained using Kuang’s formulation [8] including the Hertzian contact stiffness as a constant term for each angular position [3]. Meshing damping coefficients are taken to be proportional to the corresponding stiffness if teeth are in contact otherwise the lubricant squeeze effect is considered.

The pressure distribution around the gears is calculated simultaneously solving the continuity equations for each isolated space from the low to the high pressure chamber. In order to define the flows between each volume, the gear centre position relative to the case must be known as well as gear geometry and fluid characteristics. Then, an ordinary differential equation system must be
solved obtaining the pressure in a vane around a whole turn of the gear (for more details see [1], [5] and [6]). Due to manufacturing tolerances, relief grooves connect input and output chambers during a short period of time. So it was assumed that there is no pressure rise in the trapped volume between teeth when this volume passes from the high to the low pressure chamber, assuming a linear pressure transition. Pressure calculation includes also the case inner profile defined during a wear procedure that takes place at the end of the manufacturing process [2]. It was also taken into account the displacement toward the low pressure chamber of the lateral floating brushes. Once the pressure value for each gear angular position is known, it is possible to obtain the resultant pressure forces ($f_{pX0}, f_{pY1}$) and torque ($M_{p}$) on each gear as a function of its centre position relative to the case. The non-linear forces of the hydrodynamic journal bearings are implemented using the formulation proposed by Childs et al. [7], called “finite impedance formulation”.

As described, the model is highly non-linear. The dynamic equations of motion for each degree of freedom are obtained in [3, 4] and are numerically integrated in SIMULINK environment. With the aim to reduce the integration time, the average positions of the shaft axes inside the journal bearings are estimated before the integration of the dynamic model. This estimation is carried out setting the periodically variable pressure and meshing forces of the model to a constant value equal to their mean values. Then the ‘stationary’ axis positions (orbit centroid) are computed, as the solution of a non-linear system of algebraic equations obtained from the force balance of each gear. A flow chart of the computational procedure used for calculating the orbit centroid position is shown in Figure 3. More details about the pressure distribution and model dynamics can be found in [1-4].

![Figure 2: (a) Reference Frame and (b) Model Scheme](image-url)
The model provides as output the position, velocity and acceleration for gear 1 and 2 as well as bearing, meshing and pressure forces. The results about simulation show that the model serves to increase the knowledge about the relative importance of each phenomenon implied in the dynamic behaviour and consequently on the vibration and noise generation. The more relevant events in the dynamic response are those related with the discontinuities in the pressure forces and torques, due to the change in the number of isolated volumes carrying fluid and to the trapped volume [2, 3].

3 Test Set-up and Validation Procedure

Experimental tests were carried out in a gear pump test bench designed specifically with the aim to achieve a good characterization about noise and vibration of each unit under test (Figure 4a). The test bench is available at TRW Automotive Italia SpA – Divisione Automotive Pumps. The pump under test is fastened on an ergal plate that also provides proper connections to low and high-pressure oil pipes. The pump is driven by an electrical motor with inverter by means of an Oldham coupling. The electrical motor is equipped by an encoder that gives 125 pulses per revolution. Test bench operation is controlled by a specific application developed in Labview allowing gear rotational speed until 5000 r.p.m. and pressure ranges up to 110 bar. The plate is equipped with four high-impedance quartz-based triaxial force sensors (Kistler 9251, frequency range 30 kHz) and four piezoelectric accelerometers (PCB 353B18, frequency range 1 Hz to 10 kHz), see Figure 4.
reference frame considered for the plate is shown in Fig. 4b: the origin is on the axis of the driving
shaft (centre of gear 1, O1) and Z axis is downward (coordinate \( \theta \) is clockwise). Figure 2(a) also
shows the relationships between these reference frame of the plate (at which the pump case is
fastened) – called RFC – and the reference frames of gears 1 and 2 – called RF1 and RF2,
respectively. Force sensors allow the measurement of six reaction components acting on the plate:
three ortogonal X, Y and Z force components and three torques about each individual axis, while
accelerometers provide information about the plate accelerations on the plane XY: the positive
directions of the measured accelerations are \(-Y, +Y, +X, -X\) for accelerometers A, B, C, D,
respectively. Eight channels (4 forces and 4 accelerations) are acquired and processed by a LMS
Scadas SC305 Front-end controlled by software LMS Test-Lab.

![Figure 4: (a) Pump under test ;(b) Force plate: locations of force sensors and accelerometers: 1, 2, 3,
4 – force sensors; A, B, C, D – accelerometers; P – high- pressure pipe .](image)

Model validation is not a simple task in complex systems like gear pumps where it is not easy to
directly obtained vibration data concerning rotating components. Furthemore, it is necessary to
acquire secondary measurements that, after proper processing, provide quantities that can be
compared with simulation results. In the present case, D’Alembert’s equations of the system are
considered, where some terms are obtained by means of measurements and others are simulation
results. D’Alembert’s principle is applied to the whole system composed of the pump and the force
plate, considering the pump case rigidly fixed to the plate. Only forces and moments in the plane
orthogonal to the gear axes are taken into account. Using the reference frame of the force plate
XO1Y shown in Figures 2(a) and 4(b) with the origin at the centre O1 of gear 1 and evaluating
moments about point O1, D’Alembert’s equations are:

\[
\begin{align*}
\sum F_x + \sum F_{y1} + \sum F_{z1} &= 0 \\
\sum F_x + \sum F_{y2} + \sum F_{z2} &= 0 \\
\sum M_{x1} + \sum M_{y1} + \sum M_{z1} &= 0
\end{align*}
\]

(1 –3)

where the external force components are (see the list of symbols in Appendix A):

\[
\begin{align*}
\sum F_x &= R_{x12} + R_{x34} \\
\sum F_y &= R_{y14} + R_{y23} + F_p \\
\sum M_{x1} &= (R_{x12} - R_{x34}) \frac{h_1}{2} + (R_{y23} - R_{y14}) \frac{h_2}{2} - F_p \cdot b_p + M_s
\end{align*}
\]

(4 –6)
the inertia components concerning gears are first expressed in the reference frames of the gears and then transformed to the reference frame of the force plate:

\[
\sum F_{igx} = -\left(m_1\ddot{x}_1 + m_2\ddot{x}_2\right)\sin\alpha - \left(m_1\ddot{y}_1 + m_2\ddot{y}_2\right)\cos\alpha
\]

\[
\sum F_{igy} = -\left(m_1\ddot{x}_1 + m_2\ddot{x}_2\right)\cos\alpha + \left(m_1\ddot{y}_1 + m_2\ddot{y}_2\right)\sin\alpha
\]

\[
\sum M_{igx} = -(\alpha)(-m_1\ddot{x}_1 \sin\alpha - m_2\ddot{y}_2 \cos\alpha) - J_1\ddot{\theta}_1 + J_2\ddot{\theta}_2 \quad (7-9)
\]

and the inertia components concerning the pump case and force plate are:

\[
\sum F_{icx} = -m_{pp}\ddot{x}_G = -m_{pp}\left(\ddot{x}_G - \ddot{\theta} y_G\right) = -m_{pp}\left(\ddot{x}_C - \ddot{\theta} y_G - \ddot{x}_P - \ddot{\theta} y_P\right)
\]

\[
\sum F_{icy} = -m_{pp}\ddot{y}_G = -m_{pp}\left(\ddot{y}_G + \ddot{\theta} x_G\right) = -m_{pp}\left(\ddot{y}_C + \ddot{\theta} x_G + \ddot{y}_P + \ddot{\theta} x_P\right)
\]

\[
\sum M_{ico} = -J_0\ddot{\theta} - F_{icx} y_G + F_{ict} x_G = -J_0\ddot{x}_G + \ddot{\theta} + F_{icx} y_G + F_{ict} x_G \quad (10-12)
\]

While the gear inertia components are given by the simulation results in terms of gear accelerations, the inertia components concerning pump case and force plate can be evaluated by acceleration and force measurements. Regarding the external components, sensor reactions are measured, the force due to the high-pressure pipe, \(F_P\), can be experimental evaluated and the driving shaft torque, \(M_S\), is given by simulations. It is worth noting that both accelerometers and force sensors have a low frequency limit and do not measure the continuous component. As a consequence, the mean values of all the quantities have to be cancelled, in order to properly compare simulation and test results. Thus, the mean value of the driving shaft torque, \(M_S\), is cancelled and the high-pressure pipe, \(F_P\), is not considered, since the pressure variation around the mean value gives a negligible contribution with respect the other components.

Thus, reorganizing Eqs. (13-15) in order to put the terms given by measurements on the left side and the terms given by simulation on the right side, the following equations are obtained:

\[
R_{x12} + R_{x34} + \sum F_{icx} = -\sum F_{igx}
\]

\[
R_{y14} + R_{y23} + \sum F_{icy} = -\sum F_{igy}
\]

\[
(R_{x12} - R_{x34})\frac{b_2}{2} + (R_{y23} - R_{y14})\frac{b_1}{2} + \sum M_{ico} = -\sum M_{igx} - M_S \quad (13-15)
\]

It is worth noting that the terms on the right side of these equations are the opposite of the inertia and external components directly acting on the gears; these components load the pump case and the plate and produce the reactions and the inertia forces on the left side. In operation conditions these are the components that can excite case vibrations and produce noise. Consequently, their estimation has a practical interest.

In the following section the validation is carried out comparing the experimental data on the left side of these equations with simulation ones on the right side.

4 Results and Discussion

The validation is carried out using data both in time and in frequency domain. The model parameters were preliminarily evaluated on the basis of both design and literature data; details are given in Refs. [1, 3]. The values of damping factors \(\gamma_T\) and \(\gamma_m\) (proportionality factor between damper constant and stiffness for the driving shaft and the tooth meshing, respectively) are adjusted in order to better match experimental results. Table 1 lists the values of the model parameters.
The validation results in time domain concern operational pressure of 34 and 90 bar and angular speed of 2000 and 3350 rpm. The time histories of the experimental forces and accelerations on the left side of Eqs. (13-15) were acquired using sampling frequency of 51.2 kHz and frequency resolution 1.56 Hz, while the gear accelerations and the shaft torque were computed by means of the model for a time corresponding to 48 meshing periods \( T \), using display frequency of 48 kHz. Since the available accelerometers data have upper frequency limit of 10 kHz, both experimental and simulation quantities were digitally filtered at 10 kHz. The time synchronous average (TSA) of the experimental quantity on the left side of Eqs. (13-15) was computed over 48 meshing periods at 2000 rpm and 400 meshing periods at 3350 rpm, while the TSA of the simulation quantity on the right side was computed over 48 meshing periods. Figure 5 shows the comparison between the experimental RMS values of the TSA and the simulation ones. The agreement between the RMS values is not satisfactory at 3350 rpm - 90 bar in the X-direction and at 3350 rpm - 34 bar in the Y-direction, but in the all other operational conditions and directions the agreement is rather good: this indicates that the model is able to give a satisfactory evaluation of the force levels exciting the case.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 )</td>
<td>0.040 kg</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>0.021 kg</td>
</tr>
<tr>
<td>( J_1 )</td>
<td>4.2 x 10^{-7} ( \text{kg m}^2 )</td>
</tr>
<tr>
<td>( J_2 )</td>
<td>3.9 x 10^{-7} ( \text{kg m}^2 )</td>
</tr>
<tr>
<td>( K_f )</td>
<td>1.06 - 1.4 x 10^8 ( \text{N/m} )</td>
</tr>
<tr>
<td>( K_T )</td>
<td>5.5 x 10^5 ( \text{N m/rad} )</td>
</tr>
<tr>
<td>( \tau_f )</td>
<td>10^{-9} s</td>
</tr>
<tr>
<td>( \gamma_m )</td>
<td>10^{-10} s</td>
</tr>
</tbody>
</table>

Table 1 – Model parameters

Figure 5: RMS values of the TSA relative to one meshing period of (a) force in X-direction, (b) force in Y-direction, (c) momentum.

The validation results in frequency domain concern two linear run-up tests from 2000 to 3350 rpm, with operational pressure of 34 bar and 90 bar respectively, conducted with the apparatus described in the previous section. The run-ups were acquired using sampling frequency of 25.6 kHz and frequency resolution of 3.125 Hz. During the run-up tests were acquired 67 spectra of the quantity...
on the left side of Eqs. (13-15). These spectra are taken at speed intervals of 20 rpm between 2000 to 3350 rpm and they are shown as waterfall maps in Figures 6 to 8. In order to compare simulation data with the experimental run-ups, 27 simulations were conducted at operational pressure of 34 bar and 90 bar and angular speed interval of 50 rpm from 2000 to 3350 rpm. The simulations were carried out for a time corresponding to 48 meshing periods \( T \), using display frequency of 48 kHz. For each simulation the quantity on the right side of Eqs. (13-15) was evaluated and its frequency spectrum was calculated. Figures 6 to 8 show these amplitude spectra as waterfall maps.

As mention the model is non linear and natural frequencies could not be defined. However it can be useful to evaluate the natural frequencies and mode shapes of an undamped linearized model, in order to compare these natural frequencies with the experimental resonances exhibited in the waterfall maps. With the aim to obtain the conventional matrix formulation with masses and stiffnesses, the variable meshing stiffness is replaced by its mean value, while the bearing forces are linearized by the expansion in a Taylor series about the stationary axis position \((x_{js}, y_{js})\), giving a bearing stiffness matrix. It worth noting that this stationary axis position, and consequently the natural frequencies depend on the operational conditions. Tables 2 and 3 show the natural frequencies at different operational conditions estimated by the undamped linearized model.

![Table 2 – Natural Frequencies of the linearized model at 34 bar](image)

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>34 bar - 2000 rpm</th>
<th>34 bar - 2600 rpm</th>
<th>34 bar - 3200 rpm</th>
<th>34 bar - 3350 rpm</th>
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<tr>
<td>( f_{n1} )</td>
<td>3366</td>
<td>3387</td>
<td>3408</td>
<td>3413</td>
</tr>
<tr>
<td>( f_{n2} )</td>
<td>4130</td>
<td>4130</td>
<td>4130</td>
<td>4130</td>
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<tr>
<td>( f_{n3} )</td>
<td>5001</td>
<td>5031</td>
<td>5061</td>
<td>5069</td>
</tr>
</tbody>
</table>

![Table 3 – Natural Frequencies of the linearized model at 90 bar](image)

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>90 bar - 2000 rpm</th>
<th>90 bar - 2600 rpm</th>
<th>90 bar - 3200 rpm</th>
<th>90 bar - 3350 rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{n1} )</td>
<td>4130</td>
<td>4130</td>
<td>4130</td>
<td>4130</td>
</tr>
<tr>
<td>( f_{n2} )</td>
<td>5415</td>
<td>5427</td>
<td>5439</td>
<td>5442</td>
</tr>
<tr>
<td>( f_{n3} )</td>
<td>8047</td>
<td>8065</td>
<td>8083</td>
<td>8087</td>
</tr>
</tbody>
</table>

The experimental waterfall maps (Figures 6 to 8) show some resonance regions: from about 500 Hz to 2.5 kHz in X and Y-directions and at about 3.4 kHz, 4.2 kHz, 5.2 kHz, 8 kHz and 10 kHz in all the components. The last two resonances can be identified as the natural frequencies of the whole system composed of the pump and the force plate supported by the force sensors (Kistler 9251). In fact, considering that the sensor stiffness is \( 1000 \text{ N/m} \) in X and Y-directions of Figure 4b, the system natural frequencies are 8.281 kHz for the vibration modes in the X and Y-directions and 10.675 kHz for the rotational mode (\( \theta \)-direction). This frequencies can not be obviously found in the model results because they concern a system not included in the model.

At the operational pressure of 34 bar, the experimental resonances at about 3.4 kHz, 4.2 kHz, 5.2 kHz satisfactorily agree with the natural frequencies estimated by the linearized model (Table 2). At high pressure (90 bar) the first two natural frequencies of the model (Table 3) can be found in the experimental waterfall maps, while the third natural frequency at about 8 kHz is in the same region as one of the natural frequencies of the whole system pump-force plate, thus they cannot be distinguished.
For the comparison between the experimental and simulation spectral maps, presented in Figures 6 to 8, it worth noting that they have different resolution: the frequency resolution is 3.125 Hz for experimental spectra and for the simulation ones ranges from 8.3 to 14 Hz; the resolution in the
rpm-axis is 20 rpm for the experimental spectra and 50 rpm for the simulation ones. The amplitude grey scale is the same for the pairs of maps relative to the same direction and operational conditions. As previous mention, the resonance regions at about 8 kHz and 10 kHz cannot be found in the simulation maps. In the other frequency ranges, there is a quite good correspondence in the X-direction maps at both pressure levels and in the Y-direction maps at 34 bar. The correspondence is good in the momentum maps, especially around the resonances at about 4.2 kHz.

Further research is needed in order to understand the causes of the discrepancies (especially in Y-direction at high pressure) and to adjust the model. The discrepancies between experimental and simulation results could be also due to the approximations introduced in Eqs. (13-15), when it has been considered that the pump case is rigidly fixed to the plate and that the pressure variation are negligible.

5 Conclusions

This work concerns the experimental validation of a kineto-elatodymanic model of external gear pumps. It is a non-linear lumped-parameter model, which takes into account the variability of the pressure distribution on gears, the hydrodynamic bearing behaviour, the parametric excitation due to gear meshing and tooth profile errors, the effects of the backlash between meshing teeth, the lubricant squeeze and the possibility of tooth contact on both lines of action. In order to reduce the numerical integration time, the pressure distribution is preliminary estimated for the ‘stationary’ position of the gears. This non-linear model includes all these important dynamic effects, in order to take into account and analyse their interactions.

Moreover, an experimental apparatus has been set up for the measurements of the accelerations and the force components applied to the pump case in operation conditions. The validation procedure is based on the comparison between simulation and experimental results concerning forces and moments: it deals with the external and inertia components acting on the gears, estimated by the model, and the reactions and inertia components on the test plate and the case, obtained by means of measurements. These are the components that can excite case vibrations and produce noise; consequently, their estimation has a practical interest for noise reduction.

The validation results concerning the RMS values of the time synchronous average can be considering quite good; this indicates that the model is able to give a satisfactory evaluation of the force levels exciting the case. The validation results in the frequency domain was carried out by comparing the resonances region exhibited by the experimental waterfall maps with the natural frequency of the linearized model and the simulation waterfall maps. The first natural frequencies of the linearized model can be found in the experimental maps, while the correspondence between the experimental and simulation maps is good in some cases and worse in others.

Considering all the validation results, the correspondence between simulations and tests can be globally considered satisfactory, taking into account the complexity of the model including several non-linear dynamic phenomena.

6 Acknowledgements

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References


Appendix A: list of symbols

\[ \begin{align*}
\alpha & \quad \text{operation pressure angle.} \\
\gamma_T & \quad \text{damping factor for driving shaft.} \\
\gamma_m & \quad \text{damping factor for tooth meshing.} \\
\theta_0 & \quad \text{rotational coordinate of drive input.} \\
\rho_i & \quad \text{base radius of gear } i. \\
f_{bxi} & \quad \text{force in the oil bearings in direction } X_i \text{ of gear } i. \\
f_{bys} & \quad \text{force in the oil bearings in direction } Y_i \text{ of gear } i. \\
f_{pxi} & \quad \text{pressure force in direction } X_i \text{ of gear } i. \\
f_{pyi} & \quad \text{pressure force in direction } Y_i \text{ of gear } i. \\
F_P & \quad \text{high-pressure pipe force.} \\
J_i & \quad \text{moment of inertia of gear } i. \\
J_G & \quad \text{moment of inertia of pump and plate with respect to the center of mass.} \\
K_{j} & \quad \text{stiffness of tooth pair } j. \\
K_T & \quad \text{torsional stiffness of the driving shaft.} \\
m_i & \quad \text{mass of gear } i. \\
m_{pp} & \quad \text{mass of pump and plate.} \\
M_S & \quad \text{driving shaft torque.} \\
M_{pi} & \quad \text{pressure torque of gear } i. \\
R_{X12} & \quad \text{sum of the force sensor reactions in } X \text{ direction concerning sensors 1 and 2.} \\
R_{X34} & \quad \text{sum of the force sensor reactions in } X \text{ direction concerning sensors 3 and 4.} \\
R_{Y12} & \quad \text{sum of the force sensor reactions in } Y \text{ direction concerning sensors 1 and 2.} \\
R_{Y34} & \quad \text{sum of the force sensor reactions in } Y \text{ direction concerning sensors 3 and 4.} \\
T & \quad \text{meshing period (60/nZ; } n \text{ is the gear velocity in rpm).} \\
x_{O_1}, y_{O_1} & \quad \text{stationary axis position.} \\
x_G, y_G & \quad \text{center of mass of pump and plate.} \\
\ddot{x}_C, \ddot{x}_D & \quad \text{C and D accelerometer accelerations.} \\
\ddot{x}_A, \ddot{y}_A & \quad \text{plate acceleration in point } O_i \text{ in direction } X \text{ and } Y. \\
\ddot{y}_A, \ddot{y}_B & \quad \text{A and B accelerometer accelerations.} \\
Z_i & \quad \text{number of teeth of gear } i. \\
\end{align*} \]

Subscripts

\[ \begin{align*}
 i & = 1, 2 \quad \text{denotes gears} \\
 j & = a, b, c, d \quad \text{denotes pairs of teeth} \\
\end{align*} \]