A kineto-elastodynamic model of a mechanism for automatic machine

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Abstract: This work deals with the kineto-elastodynamic modelling of a mechanism for rocking motion with a cam and timing belt drives, used in a high-performance automatic packaging machine. A lumped-parameter model is presented, having five degrees of freedom and taking account of non-linearities. Equations of motion are numerically integrated, using the finite difference method. Model parameters are evaluated on the basis of design data and measurements of the actual motion of the output link. Finally the model is satisfactorily validated by comparison with experimental results. The model makes it possible to relate the design and functional parameters of the mechanism to its dynamic behaviour; thus it is used to detect the causes of the dynamic phenomena that reduce performances.

Keywords: Automatic machine, Cam mechanism, Timing belt drive, Kineto-elastodynamic analysis, Lumped-parameter model.

Introduction

It is well-known that the dynamic analysis of mechanisms operating at high speed cannot neglect the effects of link elastic flexibility and mass distribution, as well as the effects of backlashes and friction in joints. In fact these effects may affect the dynamic response - in particular the output link motion - so deeply that mechanisms may fail to perform their tasks adequately. Moreover high accelerations and dynamic stress levels may occur, causing early fatigue failure. Finally, high vibration and noise levels may arise. Researches on modelling and dynamic response of flexible mechanisms are recently reviewed in [1] and [2]. In order to build an effective, but simple model, the main difficulties are the proper choice of the number of degrees of freedom, presence of non-linearities, estimation of the model parameters and choice of the solution method.

In the specific field of automatic packaging machines, ever increasing performances - in terms of high speed, high accuracy of product manipulation and low noise - are required. Consequently increasing attention is addressed to the kineto-elastodynamic analysis, as a tool for design optimization, as well as for fault diagnostics. In this context, this paper concerns a high-performance automatic packaging machine, composed of sub-assemblies, each containing a cam or other mechanism for reciprocal or intermittent motion, performing a specific task. In particular, the present paper presents a kineto-elastodynamic model of one of these sub-assemblies, containing a cam mechanism for rocking motion and timing belt drives. This work is thus a step towards the establishment of know-how in the field of dynamic phenomena and modelling methods for mechanism typologies most commonly used in automatic machines [3-6].

Description of the model

The mechanism (schematically shown in Fig. 1) is made up of a brushless electrical motor, a series of three timing belt drives with a global speed reducing ratio of 1/6, a cam mechanism for rocking motion with dwells, another timing belt drive with speed increasing ratio \( \tau_b=3 \), and a rocker, that is the output link acting on the product. A flywheel is fitted on the camshaft. The cam mechanism is made up of a double-disk yoke cam and an oscillating dual roller follower, so that backlash is eliminated. The cam curve is composed of two modified trapezoidal curves in the rise and a third one in the return, separated by dwells [see Figs. 5(a) and 6(a)].

A lumped-parameter model was obtained by applying the component element method [6, 7] and taking non-linearities into account, namely, backlashes, stiffness variability and Coulomb friction. The model (Fig. 2) has five torsional degrees of freedom and describes the whole mechanical system: the input is the motor shaft rotation and the output is the motion of the extremity of the rocker. The areas labelled 1 to 5 in the mechanism scheme of Fig. 1 enclose the components which have been considered lumped together in the computation of the model moments of inertia \( J_1 \) to \( J_5 \), respectively.

With reference to Figs. 1 and 2, the coordinate \( \alpha_0 \) is the known model input and represents the angular displacement of the motor shaft, reduced to the camshaft axis. It is a linear function of the time, as the motor shaft can be assumed to rotate at constant speed. The moment of inertia \( J_2 \) is obtained lumping the moments of inertia of the camshaft together with the preponderant moments of inertia of the pulley and the flywheel fitted on the camshaft itself. Thus the coordinate \( \alpha_2 \), associated to \( J_2 \), represents the pulley and flywheel angular displacement. The coordinates \( \alpha_0 \) and \( \alpha_2 \) are separated by the timing belt transmission between the motor shaft and the camshaft. Since this transmission is quite complex, accurate modelling would need to use several degrees of freedom. However, a simple model is obviously desirable; moreover previous results [1, 4] show that a good dynamic simulation is obtainable also by a simple model. Thus, the transmission between the motor and the camshaft is modelled by only one inertial element (moment of inertia \( J_1 \), with angular coordinate \( \alpha_1 \)) connected to the...
neighboring inertial elements by linear spring-damper elements, in order to globally take into account the inertial, elastic and damping properties of this transmission. The validity of this choice will be experimentally verified. All the model parameters relative to the mechanical transmission driving the camshaft are reduced to the axis of the camshaft itself.

The coordinate $\beta_2$ is obtained reducing $\alpha_2$ to the cam follower axis, using the cam curve function, $\hat{\beta}_2 = \beta_2(\alpha_2)$; so the link between these coordinates is represented by a template profile in the scheme of Fig. 2. The moment of inertia $J_3$ is obtained lumping the moments of inertia of the cam follower and its shaft together with the preponderant moment of inertia of the pulley fitted on the cam follower shaft itself; a portion of the equivalent moment of inertia of the timing belt is also added. So the coordinate $\beta_3$ represents the angular displacement of this pulley. The torsional stiffness $k_3$, reduced to the cam follower axis, is obtained by composing in series the torsional and bending stiffnesses of the cam and follower shafts, the stiffness of the Hertzian contact between cam and roller and the bending stiffness of the follower. Stiffness $k_3$ is variable due to several reasons: firstly, depending on the direction of the transmitted torque in the cam mechanism, one or the other of the two conjugate cams and corresponding follower levers are actually loaded and therefore the value of the torsional and bending stiffness is different; secondly, the Hertzian contact stiffness depends on the contact force [8] and must be evaluated instantaneously; finally, the coupling ratio between the two axes of the cam mechanism, $\tau = \frac{d\beta_2}{d\alpha_2}$, is a function of $\alpha_2$, i.e. it depends on the mechanism’s position.

Coordinate $\gamma_3$ is obtained reducing angular displacement $\beta_3$ to the rocker axis (see Fig. 1), using the timing belt drive speed ratio, $\gamma_3 = \frac{\tau_b}{\tau} \beta_3$. The moment of inertia $J_4$ is equivalent to the inertia of the pulley fitted on the rocker shaft and portions of the shaft itself and the belt; $J_5$ is the moment of inertia of the other portion of this shaft and rocker. Thus the coordinate $\gamma_4$ represents the angular displacement of the pulley fitted on the rocker shaft, while $\gamma_5$ is considered to be the tangential displacement of a point located next to the rocker extremity, at radius $R_a = 0.1075$ m from the rotation axis, reduced to angular displacement about the rocker axis. The tangential acceleration of this point was also measured for model validation. Coordinate $\gamma_3$ is the model output, as it represents the motion of the rocker part acting on the product. The effect of the longitudinal stiffness of the timing belt is taken into account with the equivalent torsional stiffness $k_4$, reduced to the rocker axis. Torsional stiffness $k_5$ is the composition to the torsional stiffness of the rocker shaft and the bending stiffness of the rocker reduced to its axis.

A viscous damper is associated to each stiffness in the model, in order to globally take account of structural damping as well as other damping. The coefficients of the dampers are taken proportional to the corresponding stiffnesses: $c_i = q_k k_i$, $i=1,...,5$. Moreover, the Coulomb friction forces acting both in the rolling bearings and in the lip seals may be taken into account applying the Coulomb friction torques, $M_{f_i}$ ($i=1,...,4$), to the first four inertial elements of the model.

In the mechanism under investigation the torque transmitted by the timing belt drives fluctuates and changes direction, due to the rocking motion and the preponderant inertial load. Thus, the side of contact between pulley and belt teeth may change, due to the backlash. Moreover, during the backlash traversing the
torque is transmitted by the Coulomb friction forces between the top land of the pulley teeth and the belt grooves (or the contrary, according to the belt type). A dynamic model of timing belt drives transmitting fluctuating torques was presented by Karolev and Gold [9]. However, in order to account for this pulley-belt interaction without excessively complicating the model, an angular backlash is introduced in parallel to the spring-damper elements representing timing belt transmissions, i.e. the first, the second and the fourth element (g1, g2 e g4 in Fig. 2). This backlash takes account of backlashes between the belt and both pulleys. During the backlash traversing, a Coulomb friction interaction is considered, both in sliding and static conditions. The maximum transmissible torques in sliding condition are indicated as M_{fb i} (i=1,2,4).

Moreover, an angular backlash g3 may be introduced in the model, in order to take account of a backlash between cam and rollers. It is zero in sound conditions, but it might actually take place, due to wear on roller and cam surfaces.

The differential equations of equilibrium relative to the above described model are non linear. They are numerically integrated, using the finite difference method. In order to obtain stability of the numerical integration and good accuracy, a suitably small time step interval is taken, related to the value of the highest natural frequency of a simplified linear model. For details on the finite difference formulation and the choose of integration step interval, see [6, 7].

**Test arrangement and model parameter estimation**

Experimental tests were carried out first in order to adjust the values of the model parameters then to validate the model results at different operating speeds ranging 160 to 400 rpm. The tangential acceleration of the output link was obtained by mounting an accelerometer next to the extremity of the rocker, at a point located at distance R_a from its axis; the tangential acceleration was then reduced to angular acceleration around the rocker’s axis. The accelerometer signal was analyzed in the time and frequency domain. A one-per-cam revolution tachometer signal was also collected.

The values of the moments of inertia and stiffnesses of the model were preliminarily computed on the basis of the dimensions of the links, with the exception of the stiffness of the fourth timing belt (between the cam follower shaft and the rocker shaft), obtained experimentally. Stiffnesses and moments of inertia were then adjusted in order to better match experimental results. Since the complex transmission driving the cam mechanism is globally modeled through only one degree of freedom, the preliminary estimation of the corresponding model parameters is expected to be less accurate with respect to the model parameters relative to the areas 3 to 5 (Fig. 1). Consequently, the moments of inertia J3 to J6 were considered correct and the values of the other parameters were adjusted. In particular it was expected that moments of inertia J1 and J2 and stiffnesses k1 and k2 would be significantly adjusted, while stiffnesses k3 to k5 would be decreased to some amount, according to results of previous researches [1].

The adjusting procedure was the following. The experimental rocker acceleration was analysed in the frequency domain in the range up to 200 Hz, containing all significant components. The amplitude spectra [Figs. 3(a,b) and 4(a,b)] show, in addition to a one-per-cam revolution component and its harmonics, a first resonance around 6 Hz, a second resonance around 14-15 Hz and a wide range of resonance from about 100 to 200 Hz. In fact the spectra present amplitude peaks at 6 Hz and about 14-15 Hz that are independent of the operating speed [Fig. 3(a,b)]. Moreover, the range of resonance from 100 to 200 Hz is shown in Fig. 4(a,b), comparing the amplitude spectrum of the theoretical angular acceleration - the excitation of the system - and the amplitude spectrum of the experimental angular acceleration - the response of the system. The model parameters were then adjusted in...
order to match the three mentioned resonance ranges with the first three natural frequencies of the model. For this aim, a simplified linear model was used, obtained from the model of Fig. 2, ignoring damping, backlashes, friction and hertzian compliance, and reducing all the moments of inertia and stiffnesses to the rocker axis using a reduction coefficient \( \tau_c \) constant and equal to its average value. The modal analysis of this model indicated that the first two frequencies are quite independent of the lumped parameters concerning the areas 3 to 5 in Fig. 1, due to the fact that the first two stiffnesses are much smaller than the others and the first two moments of inertia are higher than the others. In practice, it is possible to adjust the first two natural frequencies separately from the other ones. Firstly, the third natural frequency of the model was forced within the resonance range 100 to 200 Hz, by simply reducing the stiffnesses by about 40 %. As mentioned above, the third stiffness depends on the direction of the transmitted torque in the cam mechanism; therefore - in this simplified model - two values were awarded to it: both the corresponding values of third natural frequency are included within the resonance range of the system. Finally, an optimization procedure was used to match the first and second natural frequencies: as these frequencies are prevalently affected by parameters \( k_1, k_2, J_1 \) and \( J_2 \), only these parameters were significantly modified.

The values of the proportionality constant \( q \) between damper coefficients and stiffnesses, the friction torques \( M_{l_1} \) \( (i=1,...,4) \) and \( M_{l_2} \) \( (i=1,2,4) \), and the backlashes \( g_4 \) \( g_2 \) and \( g_4 \) have been inferred through observation of the experimental acceleration. Table 1 shows the values of the main model parameters, as well as the corresponding minimum and maximum values of the natural frequencies.

**Results and discussion**

Good agreement between numerical results and experimental results was generally attained. Figures 5(a,b)
and 6(a,b) compare the experimental output angular acceleration patterns with the numerical results in the time domain, when the nominal camshaft speed is 400 and 160 rpm, respectively. The patterns correspond exactly to one machine cycle (one cam revolution) and begin at the start of the first modified trapezoidal curve. Numerical patterns satisfactorily agree with the experimental ones both in shape and maximum values. The agreement is better at high camshaft speeds (Fig. 5), while at low speeds (Fig. 5) the model is not able to describe the oscillations at high frequency (130 to 170 Hz) in the rocker acceleration, particularly at the beginning of the machine cycle and at the beginning of the third trapezoidal curve; moreover the numerical acceleration at the second trapezoidal curve is higher than the experimental one.

The causes of the deviations of actual rocker motion from the theoretical one (i.e. the cam curve) were investigated. Firstly, the interaction between the cam curve frequency components and mechanism resonance range 100 to 200 Hz produces oscillations in the rocker acceleration in that frequency range. Secondly, it is plain that these oscillations are not exactly around the theoretical value. The origin of the latter deviations has been mainly awarded to oscillations in camshaft speed in the low-frequency range up to 20 Hz, due to the first two resonances, that are mainly related to the dynamic properties of the transmission driving the cam mechanism, as mentioned above. For instance, in the case of 160 rpm nominal speed, the model simulation shows that the camshaft speed oscillates around the nominal value of ±18 % (Fig. 7). As a matter of fact, Fig. 8 reports the numerical result of a three degrees of freedom model in which the transmission driving the camshaft is taken as rigid, so that the cam shaft speed is the nominal one: in this case the oscillations at high frequencies are exactly superimposed on the theoretical acceleration pattern.

Moreover, model simulation can indicate which is the influence of parameter changes on the kineto-elastodynamic response. For example, if $J_2$ is triplicated, simulating an increase of the flywheel’s moment of inertia, then the camshaft speed oscillation around the nominal value is reduced to ±2.7 %. Obviously, the more $J_2$ increases, the more the angular acceleration pattern of output member becomes similar to the numerical result of the three degrees of freedom model.

As a second example, a backlash between cam and rollers is simulated; it may actually take place due to wear on roller and cam surfaces. Figure 9 shows the effect of 0.1 mm backlash: it produces higher acceleration values, due to the shocks taking place when the roller hits the cam surface. This occurs when the inertia force direction reverses, causing the backlash to be traversed and the contact side between rollers and cam surface to change. Therefore, this model can also be a useful tool in diagnostics, as it makes it possible to understand the relationships between acceleration pattern changes and faults.

**Conclusions**

A lumped-parameter model for the kineto-elastodynamic analysis of a sub-assembly with a cam mechanism and
Timing belt drives, used in a high-performance automatic machine, was developed and validated with comparison with the measured acceleration of the output link. Nevertheless the model has only 5 degrees of freedom, it is able to effectively simulate the mechanism behaviour at different operation speeds, as it properly takes into account the main causes of the deviations of the actual output link acceleration from the theoretical one, using also non-linear elements. In particular, oscillations of the cam speed in the low-frequency range up to 20 Hz and consequent deviations of the output link acceleration are mainly due to the dynamic properties of the belt transmission driving the cam mechanism, while the vibrations in the frequency range 100 to 200 Hz are mainly related to the dynamic properties of the cam mechanism and the driven transmission with rocking motion. As regard the timing belt drives, the model takes account for the backlash between belt and pulley teeth and Coulomb friction during the traversing of backlash. Since the model makes it possible to relate the design and operation parameters of the mechanism to its actual dynamic behaviour, it may be a useful tool both in design optimization and in fault diagnostics.

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References